

Climate change and risk management

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Surface warming

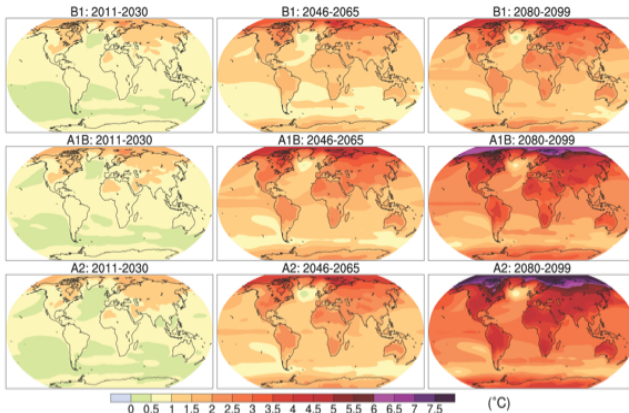


Figure 4: Projected surface temperature changes for the B1 (top), A1B (middle) and A2 (bottom) SRES scenarios averaged over the years 2011-2030 (left), 2046-2065 (middle) and 2080-2099 (right).

Some facts

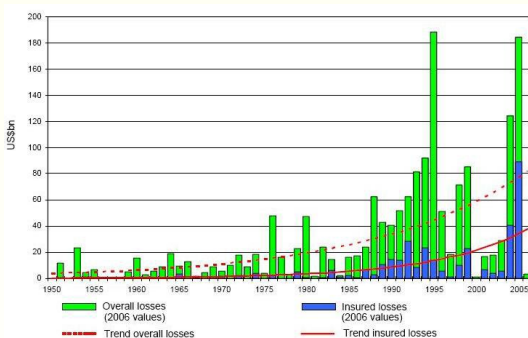


Figure 5: Major natural catastrophes on the period 1950-2005 (from Munich Re (2006).)

Natural catastrophes

Date	Loss event	Region	Overall losses	Insured losses	Fatalities
25.8.2005	Hurricane Katrina	USA	125,000	61,000	1,322
23.8.1992	Hurricane Andrew	USA	26,500	17,000	62
17.1.1994	Earthquake Northridge	USA	44,000	15,300	61
21.9.2004	Hurricane Ivan	USA, Caribbean	23,000	13,000	125
19.10.2005	Hurricane Wilma	Mexico, USA	20,000	12,400	42
20.9.2005	Hurricane Rita	USA	16,000	12,000	10
11.8.2004	Hurricane Charley	USA, Caribbean	18,000	8,000	36
26.9.1991	Typhoon Mireille	Japan	10,000	7,000	62
9.9.2004	Hurricane Frances	USA, Caribbean	12,000	6,000	39
26.12.1999	Winter storm Lothar	Europe	11,500	5,900	110

Table 1: The 10 most expensive natural catastrophes, 1950-2005 (from Munich Re (2006)).

...but also man-made catastrophes

Examples: industry fire, oil & gas explosions, aviation crashes, shipping and rail disasters, mining accidents, collapse of building or bridges, terrorism,...

Date	Location	Plant type	Event type	Loss (property)
23.10.1989	Texas, USA	petrochemical*	vapor cloud explosion	839
04.05.1988	Nevada, USA	chemical	explosion	383
05.05.1988	Louisiana, USA	refinery	vapor cloud explosion	368
14.11.1987	Texas, USA	petrochemical	vapor cloud explosion	282
07.07.1988	North sea	platform*	explosion	1,085
26.08.1992	Gulf of Mexico	platform	explosion	931
23.08.1991	North sea	concrete jacket	mechanical damage	474
24.04.1988	Brazil	plateform	blowout	421

Table 2: onshore and offshore largest property damage losses (from 1970-1999).

The largest claim is now the **9/11 terrorist attack**, with a US\$ 21,379 million insured loss.

2010: a year of devastating and costly events

- **304 catastrophic events** (167 natural catastrophes and 137 man made disasters);
- **Victims**: nearly **304 000** (more than 222 000 died in the massive earthquake that struck Haiti in January);
- **Economic losses**: close to **USD 218bn** (versus USD 68bn in 2009). Asia was the region with the highest losses because of floods of unprecedented dimensions;
- **Cost to insurers**: more than **USD 43bn** (versus USD 27bn in 2009).

Compared to 2009, insured losses were more than 60% higher in 2010, but still below 2005, the year that insured losses soared after Hurricanes Katrina, Wilma and Rita struck the US.

2010: to sum up

Region	Number	Victims	Insured loss (in USDm)	Economic losses (in USDm)	As a % of GDP
North America	36	139	15 348	20 551	0.13%
Latin America and the Caribbean	39	225 784	8 977	53 378	1.10%
Oceania/Australia	7	50	8 860	13 131	0.95%
Europe	37	56 490	6 303	35 204	0.19%
Asia	139	17 955	2 240	74 840	0.28%
Africa	32	2'640	124	337	0.02%
Seas / Space	14	515	1 623	20 623	–
World total	304	303 573	43 475	218 064	0.31%

Source: Swiss Re Economic Research & Consulting

Figure 7: Catastrophes in 2010 by region.

2010: focus on Latin America and the Caribbean

- Earthquake in **Haiti** (January) and in **Chile**;
- Hurricanes Alex and Karl in **Mexico** (causing total economic losses of more than USD 7bn and insured losses of more than USD 400m);
- Cold wave and harsh weather in **Peru, Chile and other South American countries** (causing 522 deaths);
- Two floods in **Brazil and Colombia** (causing 500 deaths);
- Tropical storm Agatha in **Guatemala and Honduras** (causing 301 deaths).

2010: what kind of devastating and costly events?

- **Earthquakes** losses accounted for almost one third of insured losses in 2010 (Chile and New-Zealand);
- **Winter storm** Xynthia in northwestern Europe led to insured losses of USD 2.8bn, killing 64 people;
- A major US **storm** caused more than USD 2bn of insured losses;
- **Floods** in Australia triggered approximately USD 2bn in claims.

Potential threats to the insurance industry

- **Property and casualty insurance**
 - Flooding - increased precipitation, rise in sea level etc.
 - Increase in frequency and severity of floods and storms
 - Pests - forestry and agriculture
 - Worldwide economic losses due to natural disasters appear to be doubling every 10 years and next decade will reach \$150 bn (Source UNEP Financial Initiatives Climate Working Group Report 2002)
- **Life and health insurance**
 - Change in tropical disease vectors
 - Mortality rate changes (e.g. - increase of respiratory diseases and allergies)
- **Potential financial areas of vulnerability?**
 - Reserves (investments and surplus)
 - Ratings and solvency

To sum up

- **Climate risks** can be understood in a general setting as risks induced by climate change:
 - health issues because of *new* diseases or resurgence of diseases that were supposed to have disappeared, e.g. dengue fever, malaria, cholera in North America;
 - impact of climate in economics, e.g. agriculture or energy.
- From **global warming** to **increasing of natural catastrophes**
- but also **human actions** partially responsible! (e.g. flood is due to rainfall but also linked to the stability of soil structures which can be influenced by human constructions)

To sum up (ctd)

- **Earthquake** fatalities and insured losses are rising because of higher population densities and because populations are growing in seismically active areas;
- **Prevention, mitigation and risk avoidance** with measures such as hazard mapping or comprehensive building codes are the most important steps for dealing with catastrophes. But not all risks can be avoided, so preparing for the financial aspects of risk is a key element of any disaster-prone country or region!
- **increasing frequency and severity** of natural catastrophes;
- Without taking into account climate change, climate risk, and more specifically natural disasters, is a challenging issue to the insurance industry, since it involves potential **extremely large losses**...

But how to define a large claim?

Teugels (1982) tried to set up **three definitions** of what one might call a “large claim” (but none of these approaches seemed to satisfy the practitioners!). Large claims are:

- 1) the upper 10% largest claims;
- 2) every claim that consumes at least 5% of the sum of claims, or at least 5% of the net premiums;
- 3) every claim for which the actuary has to go and see one of the chief members of the company.

Examples: traditional types of catastrophes, natural (hurricanes, typhoons, earthquakes, floods, tornados...), man-made (fires, explosions, business interruption...) or new risks (terrorist acts, asteroids, power outages...).

What is a catastrophe?

From **large claims** to **catastrophe**, the difference is that there is a **before** the catastrophe, and an **after**: something has changed !

Before Katrina



After Katrina

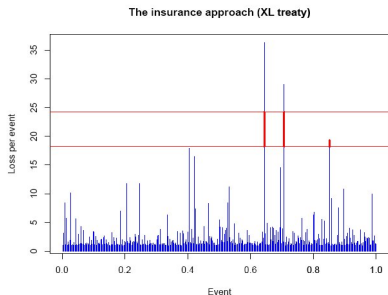
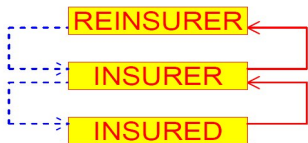


Figure 8: All state's reinsurance strategies, 2005 and 2006.

And how to face large claims?

To avoid insolvency problems, the insurance world's reaction has been the creation of a **reinsurance market**, that is to transfer the large risks to better diversified companies.

As an example, we will consider a particular type of reinsurance treaties which is the so-called **Excess of loss (XL) reinsurance** contract.



Excess of loss (XL) reinsurance

The reinsurer pays for the claim amount in excess over a given limit. Formally, let X denote the claim size and d denote the **retention level**; the **intervention of the reinsurer** concerns the random amount

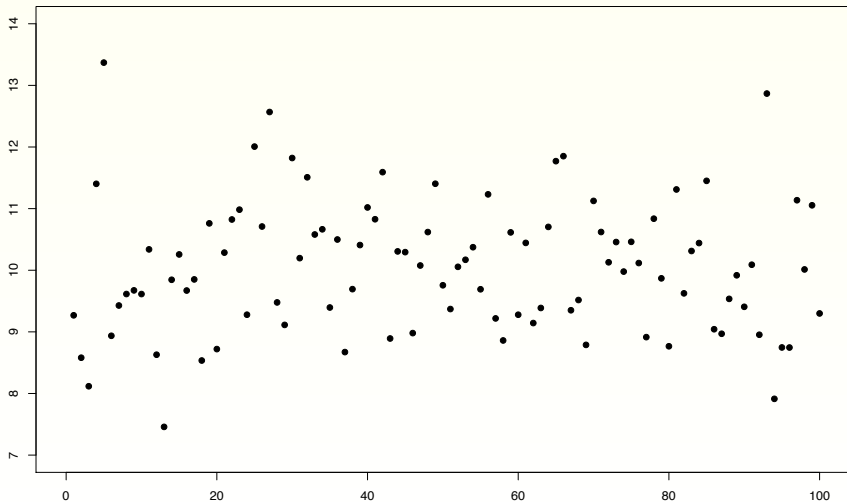
$$(X - d)_+$$

Hence, the **pure premium** is given by

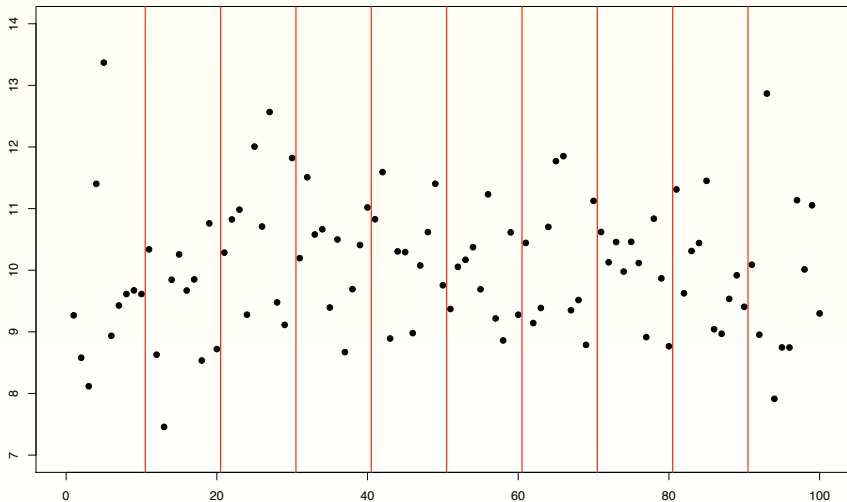
$$\pi_d = E((X - d)_+)$$

In order to estimate π_d , we need results from extreme value theory, in particular we will consider the Pickands-Balkema-de Haan's theorem.

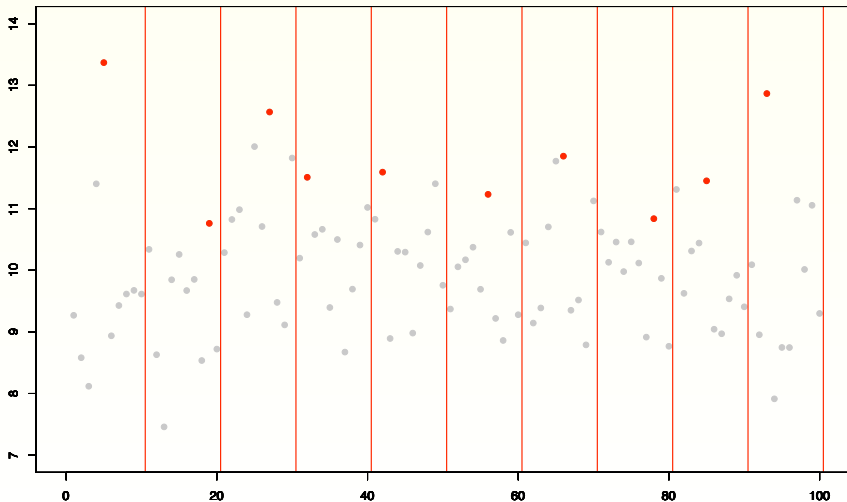
A quick reminder: univariate extreme value theory (GEV approach)



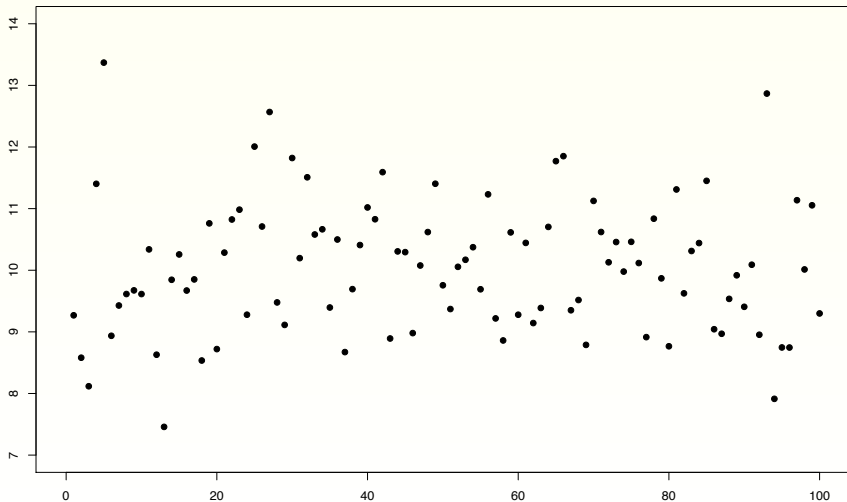
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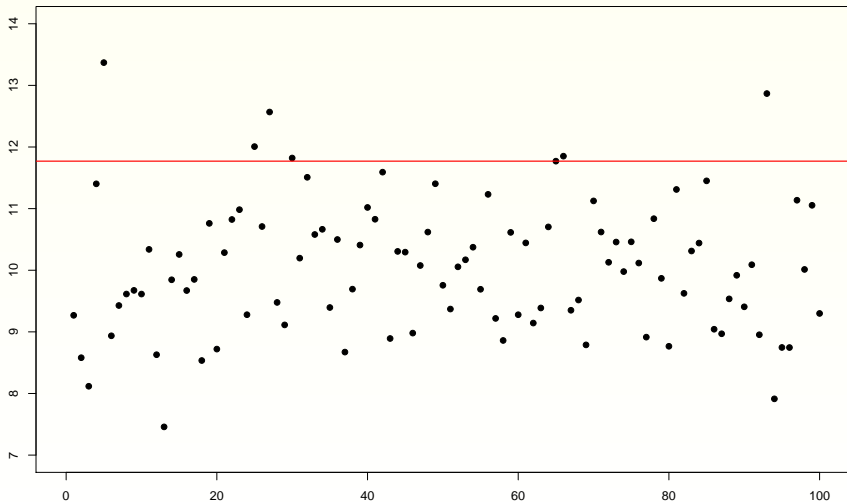
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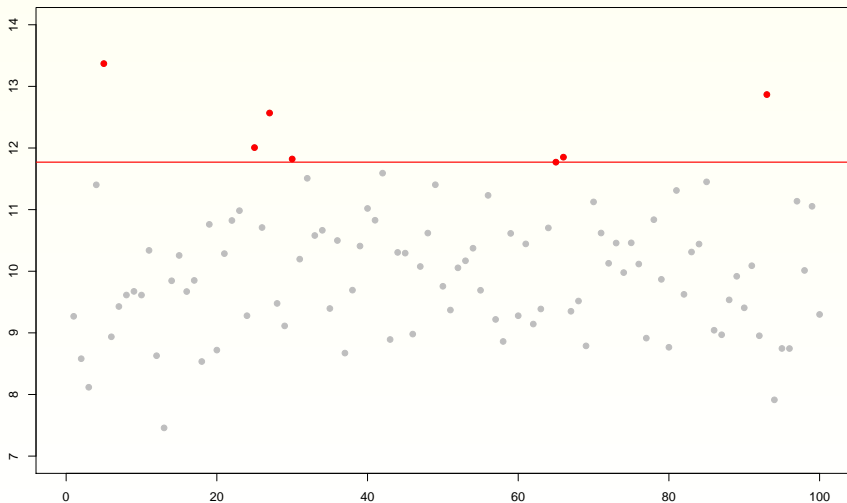
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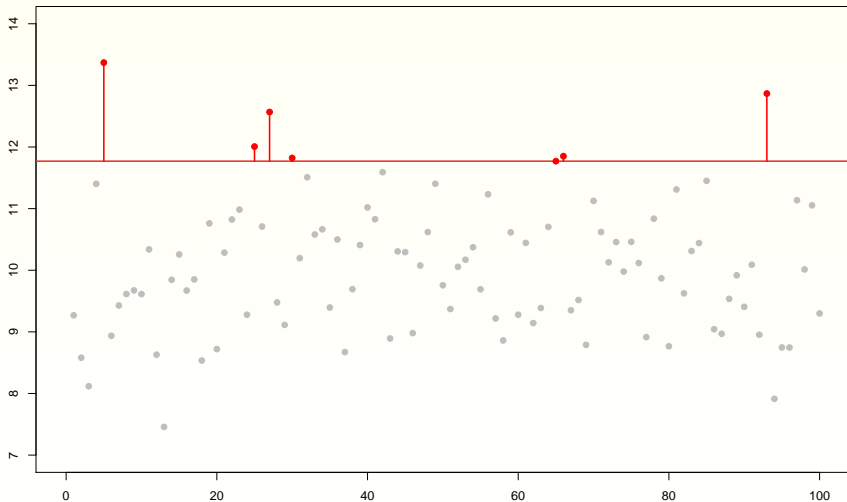
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Pickands-Balkema-de Haan's theorem

Theorem $F \in MDA(G_\xi)$ if and only if

$$\lim_{u \rightarrow x_F} \sup_{0 < x < x_F} \{|P(X - u \leq x | X > u) - H_{\xi, \sigma(u)}(x)|\} = 0$$

for some positive function $\sigma(\cdot)$, where

$$H_{\xi, \sigma}(x) = \begin{cases} 1 - (1 + \xi x / \sigma)^{-1/\xi}, & \xi \neq 0 \\ 1 - \exp(-x/\sigma), & \xi = 0 \end{cases}$$

Calculating the pure premium...

$$1 - F(x) \approx (1 - F(u))[1 - H_{\xi, \sigma(u)}(x - u)], \text{ for all } x > u$$

So, if $u = X_{n-k:n}$, then

$$1 - F(x) \approx (1 - F(X_{n-k:n}))[1 - H_{\xi, \sigma(u)}(x - X_{n-k:n})],$$

for all $x > X_{n-k:n}$, but

$$1 - F(X_{n-k:n}) \approx 1 - \hat{F}_n(X_{n-k:n}) = k/n$$

Calculating the pure premium...

Recall that $\pi_d = E((X - d)_+)$ with d large, thus,

$$\begin{aligned}\pi_d &= \frac{1}{P(X > d)} \int_d^\infty (1 - F(x)) dx \\ &\approx \frac{k}{n} \frac{\sigma}{1 - \xi} \left(1 + \xi \frac{d - X_{n-k:n}}{\sigma} \right)^{1 - \frac{1}{\xi}}\end{aligned}$$

i.e.

$$\hat{\pi}_d = \frac{k}{n} \frac{\hat{\sigma}_k}{1 - \hat{\xi}_k} \left(1 + \hat{\xi}_k \frac{d - X_{n-k:n}}{\hat{\sigma}_k} \right)^{1 - \frac{1}{\hat{\xi}_k}}$$

(see e.g. Berlaint et al. (2005))

In a similar way, it is possible to derive explicit formulas for any **tail risk measures**: VaR, TVaR,...

Value-at-Risk and Expected Shortfall

The **Value-at-Risk** is simply the quantile of a profit & loss distribution

$$VaR(X, p) = x_p = F^{-1}(p) = \inf\{x \in R, F(x) \geq p\}$$

The Expected Shortfall, or **Tail Value-at-Risk**, is the expected value above the VaR

$$TVaR(X, p) = E(X|X > VaR(X, p))$$

Estimating the Value-at-Risk

A natural idea is to use the Pareto approximation for claims exceeding threshold u .

Let N_u denote the number of claims exceeding u

$$N_u = \sum_{i=1}^n 1_{(X_i > u)}$$

If $x > u$,

$$\begin{aligned} \bar{F}(x) &= P(X > x) = P(X > u)P(X > x|X > u) \\ &= \bar{F}(u)P(X > x|X > u) \end{aligned}$$

where $P(X > x|X > u) = \bar{G}(x - u)$ is Pareto distributed

$$G(t) = P(X - u \leq t|X > u) \sim H_{\xi, \sigma}(t)$$

Estimating the Value-at-Risk

Thus, a natural estimator for $\overline{F}(x)$ uses a natural estimator for $\overline{F}(u)$, and the Pareto approximation for $\overline{F}_u(x)$, i.e.

$$\hat{F}(x) = 1 - \frac{N_u}{n} \left(1 + \hat{\xi} \frac{x - u}{\hat{\sigma}} \right)^{-1/\hat{\xi}}$$

for all $x > u$, and u large enough.

Thus, a natural estimator for $VaR(X, p)$ is

$$\widehat{VaR}_u(X, p) = u + \frac{\hat{\sigma}}{\hat{\xi}} \left(\left(\frac{n}{N_u} (1 - p) \right)^{-\hat{\xi}} - 1 \right)$$

Estimating the Expected Shortfall

Similarly, the expected shortfall can be simply estimated by

$$\widehat{ES}_u(X, p) = \widehat{VaR}_u(X, p) \left(\frac{1}{1 - \hat{\xi}} + \frac{\hat{\sigma} - \hat{\xi}_u}{[1 - \hat{\xi} \widehat{VaR}_u(X, p)]} \right)$$

Implications for risk capital requirements

Solvency margins when insuring against natural catastrophes
 → Dependence implies more volatility and therefore more capital requirement!

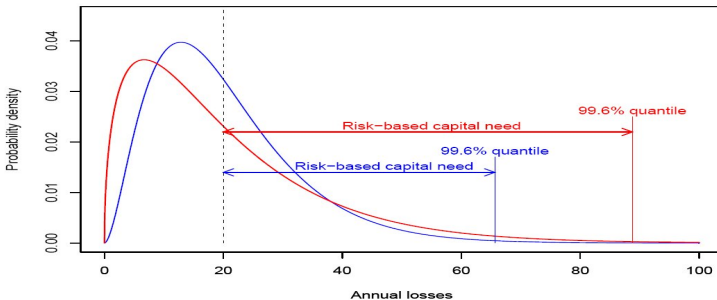


Figure 9: Independent versus dependent claims, and capital requirements.

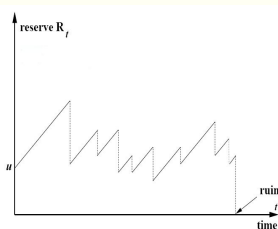
Coming back to the risk of insolvency...the ruin problem



F. Lundberg (1903)



H. Cramér



$$R_t = u + ct - \sum_{i=1}^{N(t)} X_i \quad t \geq 0$$

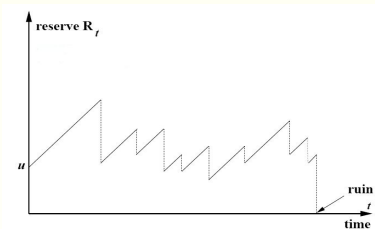
$u \geq 0$ initial capital

$c > 0$ premium income rate

X_i i -th claim size (i.i.d. r.v.s with common d.f. F)

$N(t)$ homogeneous Poisson process (λ)

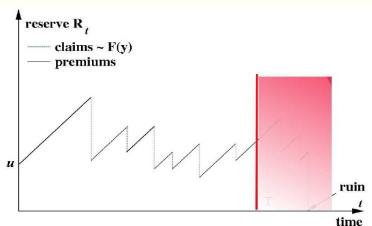
Ruin probability



The probability of **ultimate ruin**:

$$\psi(u) = P \left(\inf_{t \geq 0} R_t < 0 \mid R_0 = u \right)$$

Ruin probability (ctd.)



Finite time ruin probability:

$$\psi(u, T) = P \left(\inf_{0 \leq t \leq T} R_t < 0 \mid R_0 = u \right)$$

Classical results on ruin probabilities

- Exact solutions
- Numerical methods
- Approximations

The most celebrated result of risk theory is the **Cramér-Lundberg approximation**. For the compound Poisson model, it states that

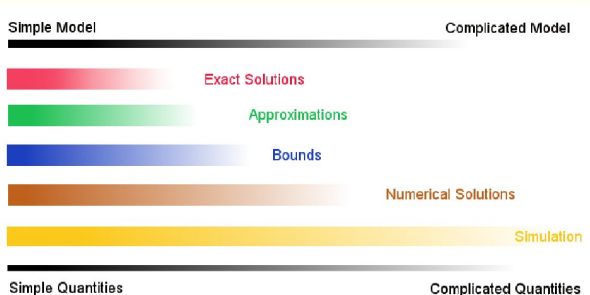
$$\Psi(x) \sim Ce^{-\nu x} \quad x \rightarrow \infty$$

where $C = (1 - \lambda\mu)/(\lambda M'(\nu) - 1)$, $\nu > 0$ is the solution of the Lundberg equation $\lambda(M(\nu) - 1) - \nu = 0$ and $M(s)$ is the moment generating function of the claim size df F .

- Bounds and inequalities

$$\Psi(x) \leq e^{-\nu x}$$

Trade-off for computability



Updating actuarial models : the heavy-tailed case

In the case of large risks or catastrophes, **claim size has heavy tails** (e.g. the variance is usually infinite), but the Poisson assumption for occurrence is still relevant.

The main result is Embrechts-Veraverbeke (1982) who gives asymptotics for the ruin probability. Really, if $F_I \in S$, then, as $u \rightarrow \infty$,

$$\Psi(x) \sim \rho^{-1} \bar{F}_I(u)$$

Modulation by a Markovian environment process

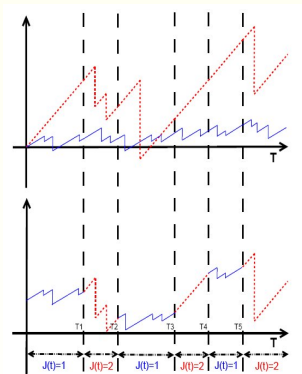


Figure: Example of a Markov-modulated risk process with **two states** (red and blue).

The multivariate ruin problem

We are interested in insurance claims which occur in d ($d > 1$)
business lines :

- activity of the company in different countries
- different types of policies offered by the company
 - house insurance
 - driving insurance
 - health
 - ...
- individual companies participating in some agreement of mutual financial support

The main question is :

how to model the dependence between business lines ?

Mathematical framework

- $\{\mathbf{Z}_k, k \geq 1\}$ are *i.i.d.* \mathbb{R}^d -valued random vectors representing claim sizes. We will assume $\mathbf{Z} \in RV(\alpha, \mu)$.
- $N_t = \#\{n \geq 1 : T_n \leq t\}$ denote the total number of claims up to time t



$$T_0 = 0, \quad T_n = W_1 + \dots + W_n$$

where $\{W_k, k \geq 1\}$ is a sequence of *i.i.d.* random variables representing the interarrival times

- $\{\mathbf{Z}_k, k \geq 1\}$ independent of $\{W_k, k \geq 1\}$

The multivariate risk process

$$\mathbf{R}_t = u\mathbf{b} + t\mathbf{p} - \sum_{k=1}^{N_t} \mathbf{Z}_k$$

- u is the initial capital
- \mathbf{b} : $\mathbf{b} \in (0, 1]^d$ and $b^{(1)} + \dots + b^{(d)} = 1$
- capital allocation to the different lines of business is determined according to $u\mathbf{b}$ ($ub^{(j)}$ is the amount of capital in business line j)
- $\mathbf{p} \in (0, \infty)^d$ is the premium income rate which is assumed to be constant

The multivariate ruin probability

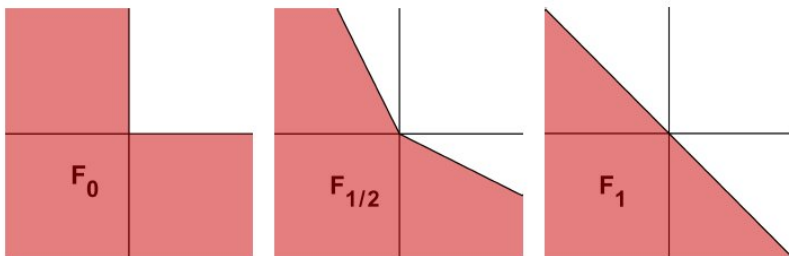
The *ruin probability*, $\psi(u)$, is the probability that the multivariate risk reserve process \mathbf{R}_t enter some domain of \mathbb{R}^d , called insolvency region (or ruin set)

$$\psi_{d,F_\beta}(u) = P(\mathbf{R}_t \in F_\beta \text{ for some } t \geq 0)$$

In the *univariate case* : $F_\beta = (-\infty, 0)$

In the *multivariate setting*, several different choices...

The insolvency region



- For $\beta = 0$, it is not possible to transfer capitals and

$$\Psi_{d,F_0} = \Psi_{or}(\mathbf{u}, T) = P(\exists j \in [1, d], \exists t^j \in [0, T], R^{(j)}(t^j) < 0)$$

- For $\beta = 1$, there is no limitation to capital transfer and

$$\Psi_{d,F_1} = \Psi_{sum}(\mathbf{u}, T) = P\left(\exists t \in [0, T], \sum_{j=1}^d R^{(j)}(t) < 0\right)$$

Poisson shock model

One type of shock may affect all lines of business (example: natural catastrophes (hurricane, ...)).

- Shocks arrive at the jump times of a Poisson process $(N_{0,t})_{t \geq 0}$ with intensity λ_0 . Thus, the part corresponding to “*common losses*” :

$$\mathbf{C}_{0,t} = \sum_{k=1}^{N_{0,t}} \mathbf{Z}_{0,k}$$

where $\mathbf{Z}_{0,k} = \mathbf{a}Z_{0,k} = (a^{(1)}Z_{0,k}, \dots, a^{(d)}Z_{0,k})$ and $a^{(j)} \geq 0$.

- Business line “*specific*” claims : such claims arrive to business line j at the jump times of a Poisson process $(N_{j,t})_{t \geq 0}$ with intensity λ_j and $Z_{j,k} = \sigma^{(j)}Z$, $j = 1, \dots, d$.

The total claim amount process

The **total claim amount process** can be written as

$$\mathbf{C}_t = \sum_{k=1}^{N_{0,t}} \mathbf{Z}_{0,k} + \sum_{k=1}^{N_{1,t}} Z_{1,k} \mathbf{e}_1 + \cdots + \sum_{k=1}^{N_{d,t}} Z_{d,k} \mathbf{e}_d = \sum_{k=1}^{N_t} \mathbf{Z}_k$$

where $(\mathbf{e}_1, \dots, \mathbf{e}_d)$ are the basis vectors in \mathbb{R}^d and

$N_t = N_{0,t} + \cdots + N_{d,t}$ is a Poisson process with intensity

$\bar{\lambda} = \lambda_0 + \cdots + \lambda_d$. Moreover

$$\mathbf{Z}_k \stackrel{d}{=} \mathbf{Z}_{0,1} \delta_0(\xi) + Z_{1,1} \mathbf{e}_1 \delta_1(\xi) + \cdots + Z_{d,1} \mathbf{e}_d \delta_d(\xi)$$

where ξ is independent of $\mathbf{Z}_{0,1}, Z_{1,1}, \dots, Z_{d,1}$ and $P(\xi = k) = \frac{\lambda_k}{\bar{\lambda}}$ for $k \in \{0, \dots, d\}$.

An approximation for the multivariate ruin probability

Let us define $\mathbf{c} = E(W)\mathbf{p} - E(\mathbf{Z})$ and assume that the net profit condition, $\mathbf{c} \in (0, \infty)^d$, holds. Assume further that $E(W^\gamma) < \infty$ for some $\gamma > \alpha$. Under these hypothesis, Hult and Lindskog (2006) state that it is possible to approximate the ruin probability of an insurance company for large initial capital u by

$$\psi_{d,F}(u) \approx \int_0^\infty \mu(v\mathbf{c} + \mathbf{b} - F) dv u P(|\mathbf{Z}| > u)$$

where μ is a measure on $\mathbb{R}^d \setminus \{0\}$ given by

$$\mu(\mathbf{z} : |\mathbf{z}| > r, \mathbf{z}/|\mathbf{z}| \in S) = r^{-\alpha} \nu(S)$$

for $r > 0$ and Borel sets $S \subset \mathbb{S}^{d-1} = \{\mathbf{x} \in \mathbb{R}^d : \|\mathbf{x}\| = 1\}$ and ν is the **spectral measure**.

An alternative formulation...

Let $\|\cdot\|$ denote any norm on \mathbb{R}^2 and let $\mathbb{S}_1 = \{\mathbf{x} \in \mathbb{R}^2 : \|\mathbf{x}\| = 1\}$ denote the unit sphere for this norm. Let \mathbf{Y} be a bivariate random vector whose distribution is *regularly varying* with tail index $\alpha > 0$, i.e. there exists a probability measure ν on the unit sphere \mathbb{S}_1 such that

$$\lim_{r \rightarrow \infty} \frac{P(\|\mathbf{Y}\| > yr, \mathbf{Y}/\|\mathbf{Y}\| \in B)}{P(\|\mathbf{Y}\| > r)} = y^{-\alpha} \nu(B)$$

for ever $y > 0$ and Borel sets $B \subset \mathbb{S}_1$ with $\nu(\partial B) = 0$.

The probability measure ν is the *spectral measure* of \mathbf{Y} (de Haan and Resnick, 1977).

Conclusions

- Insurance company are **particularly concerned by natural disaster** and the impact of those disasters has been increasing because the climate has been changing!
- The only possibility is to be able to transfer the risks either to **classical reinsurance market** or also, due to the lack of capacities, to **capital markets**.
- There is a real feeling that private insurance industry cannot continue to provide coverage against hurricanes as it has done in the past without opening itself up to the possibility of **insolvency**...